**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS=B**

**since work being 10 min after the car is dropped,the time left to complete the work is 50 mins .probability thet service manager can’t meet his commitment=P(x>50)**

**=1-pr(X<=50)**

**Convert 50 to z score.**

**Standard normal variable Z=(X-µ)/σ=(x-45)/8**

**P(X<=50)=P(Z<=(50-45)/8)=PR(Z<=0.625) =0.73237= 73.237%**

**Probability that service manager will not meet his commmitment is**

**100-73.237=26.763%=0.2676**

(1-pnorm(50,45,8)) = 0.265985529048701

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**ANS= False**

**Around 70%of the data falls within one standard deviation of the mean**

**(µ+= 38+6=44)**

from scipy import stats

from scipy.stats import norm

# p(X>44); Employees older than 44 yrs of age

1-stats.norm.cdf(44,loc=38,scale=6)

0.15865525393145707

# p(38<X<44); Employees between 38 to 44 yrs of age

stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)

0.3413447460685429

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**TRUE**

# P(X<30); Employees under 30 yrs of age

stats.norm.cdf(30,38,6)

0.09121121972586788

*# No. of employees attending training program from 400 nos. is N\*P(X<30)*

400**\***stats**.**norm**.**cdf(30,38,6)

36.484487890347154

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**== Here X1 and X2 are two independent random variables then**

**X1+X2 ~N(µ+µ,σ2 +σ2)**

**X1-X2~N(µ-µ,σ 2σ2)**

**2X1~N(2µ,2σ2)**

**2X1-(X1+X2) = N(2µ,2σ2)-N(µ+µ,σ 2+σ2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANS=D**

**The probability of getting value between a&b is 0.99**

**So,the probability of getting value outside a&b is**

**1-0.99=0.01**

**The probability towards left of a=-0.01/2=-0.05**

**The probability towards right of b=0.01/2=0.05**

**Z\*σ+µ=x**

**-(-2.57)\*20+100=151.4**

**(-2.57)\*20+100=48.6**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**ANS= Sum of two random variable having normal distribution=X**

**E(X)=E(45\*(pr1+pr2)**

**=45\*(5+7)=540**

**=SD(X)=SD(pr1+pr2)**

**=425\*(√var(pr1)+var(pr2))**

**=45\*(√9+16)**

**=225**

**X~N(540,225^2)**

**A) 95% probability for the annual profit of the company falls within two standard deviation of the mean**

**= µ2Ꝺ = 5402\*225**

**=(540+225)(540-225)**

**=(90,990)**

**qnorm(0.025,45\*5,3) # 219.1201**

**qnorm(0.975,45\*5,3) # 230.8799**

**qnorm(0.025,45\*7,3) # 309.1201**

**qnorm(0.975,45\*7,3) # 320.8799**

**The Rupee Range will be [219.12, 230.87] + [309.12, 320.87] = [528.24, 551.74]**

**B) for 5% th percentile of profit for company we use the formula**

**=µ1.5Ꝺ**

**=540-1.5\*225**

**=202.5**

**qnorm(0.05,45\*7,3) # 310.0654**

**qnorm(0.05,45\*5,3) # 220.0654**

**5th percentile of profit (in Rupees) = 310.0654+ 220.0654 = 530.1308**

**C) division1=z score of profit of zero=**

**Z=(X-µ)/Ꝺ**

**= (0-5/3)**

**=-1.66=0.0485**

> pnorm(0,5,3)

[1] 0.04779035

**Division2=Zscore of profit zero=Z(X-µ)/Ꝺ=(0-7)/4**

**=-1.75**

**=0.0401**

> pnorm(0,7,4)

[1] 0.04005916

**Division 2 has larger probability of making a loss in given year**